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VLF-LF WIDE SHIFT FSK ANTENNA FEED NETWORKS

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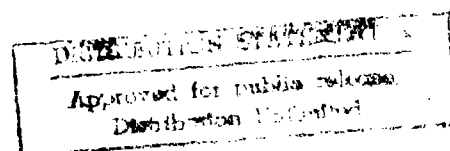
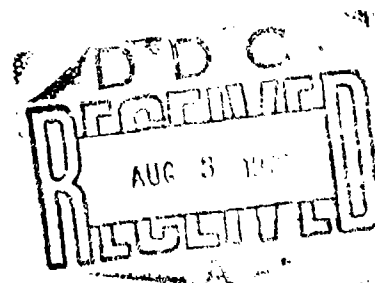
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## ABSTRACT

The problem of broadbanding VLF and LF antennas for wide shift FSK modulation has been solved by recourse to static feed networks, which exhibit either two separate passbands centered at the mark and at the space frequency respectively, or a single wide passband extending over the frequency shift. Several configurations of one-port and of two-port networks have been developed: these exhibit minimum number of elements, with values that are economically realizable. The new antenna feed networks could be applied to existing antenna installation without excessive modifications.

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## VLF - LF FSK ANTENNA FEED NETWORK DESIGN

### Introduction

The narrow bandwidths and high Q of electrically short VLF and LF antennas have limited, in general, the speed of on-off keying. By recourse to synchronous modulation of the antenna reactance (tuning inductor or added capacitor) it has been possible to extend the practical frequency shift of FSK systems to about  $\pm 50$  Hz and the word rate to about 100 per minute <sup>(1,2)</sup>. Synchronous switching, however, results in large transient phenomena, unless particular attention is paid to timing and initial conditions; for example an incremental capacitor could be switched in and out at the peak of the voltage wave (when the charging current is minimal), provided the voltage across the capacitor is maintained constant during the period while it is out of the circuit <sup>(3)</sup>. In addition, silicon controlled rectifiers exhibit severe limitations of reverse blocking voltage, and maximum forward current, necessitating recourse to expensive parallel/series combinations. An alternative approach, which is considered in this report, is based on the design of antenna feed networks, other than the conventional tuning inductor, to permit the operation of the antenna under wide frequency shift. A basic consideration in the development of such networks is that the realizations are as simple as possible and that the circuit elements, inductors and capacitors, have values which are economically realizable. It is shown in the following that a simple, practical solution is obtained by recourse to series connected one-port networks, having two separate zeros of transmission, corresponding to the two carrier frequencies of the FSK modulation. Another solution consists of the utilization of a four port network, having a Bessel type transmission response characteristics; the latter approach, which also leads to realizable networks, is based on the application of the coupled-resonator bandpass filter approximation theory or on the standard low pass/band pass approximation <sup>(4)</sup> and extends previous efforts based on the realization of Butterworth or Chebyshev filter networks <sup>(5)</sup>.

In the following, after a brief review of the basic relationships involving antenna bandwidth, FSK frequency shift, keying rate, etc, the synthesis of the antenna feed networks for wide shift FSK modulation is

presented. A frequency shift of  $\pm 250$  Hz at about 20 KHz carrier frequency will be assumed.

#### Basic Relationships in FSK antenna modulation systems

For conventional FSK transmitting antennas, with single inductor tuning, a semiempirical relationship has been developed to express the 3 dB bandwidth as a function of the frequency shift  $\pm \Delta f$ , the keying rate  $f_r$  and the keying element bandwidth product  $(\tau b)$ :

$$\begin{aligned} Bw_{3 \text{ dB}} &\approx 1.16 (\Delta f + (\tau b) f_r) \\ &= 1.16 f_r (m + (\tau b)) \end{aligned}$$

This value represents the 3 dB bandwidth required at the receiver; in the above relationship  $m = \Delta f / f_r$  is the modulation index. The keying rate  $f_r$  is proportional to the word rate  $R_w$

$$f_r = k R_w$$

where  $f_r$  is given in Hz,  $R_w$  is given in words/minute and  $k$  is a coefficient which depends on the particular encoding system:  $k = 0.19$  for Cable Morse,  $k = 0.25$  for Teletype - 5 unit synch.,  $k = 0.42$  for Teletype - 7 unit start-stop,  $k = 0.45$  for Continental Morse. For single tuned antennas the keying element bandwidth product is taken as 3 approximately; since  $\tau = 1/2f_r$  there follows  $f_r \approx b/6$  where  $b$  is the 6 dB antenna-tuning coil bandwidth. This approximate relationship provides a first order value for the maximum permissible keying rate with an antenna having 6 dB bandwidth  $b$ .

In FSK modulation it is desirable, in general, to select the modulation index so that the carrier amplitude vanishes and only the sidebands, at  $f_0 \pm n f_r$ , are present. Considering a carrier at frequency  $f_0$  which is modulated periodically and discontinuously with frequency shift from  $f_0 - \Delta f$  to  $f_0 + \Delta f$ , with a pulse of period  $T$  and keying rate  $f_r$ , i.e. with modulation index  $m = \Delta f / f_r$ , one finds that FSK modulated current may be expanded as follows:

$$i(t) = I_m \cos(\omega_0 t + \Delta \omega S(t))$$

$$= I_m \sum_{n=-\infty}^{\infty} \frac{2m}{\pi(m^2 - n^2)} \sin((m - n)\frac{1}{2}\pi) \times \cos(\omega_0 - n\omega_r)t$$

where  $S(t)$  is a square wave function (fig.1),  $\omega_0$  is the carrier frequency,  $\omega_r$  is the keying rate,  $m = \Delta\omega/\omega_r = \Delta f/f_r$ . Writing the latter expression in expanded form one has:

$$i(t) = (2I_m/\pi m) \sin(\frac{1}{2}\pi m) \cos \omega_0 t + \quad (\text{carrier})$$

$$+ \frac{2mI_m}{\pi(m^2 - 1)} \cos \frac{1}{2}\pi m \left( \cos(\omega_0 - \omega_r)t - \cos(\omega_0 + \omega_r)t \right)$$

$$- \frac{2mI_m}{\pi(m^2 - 4)} \sin \frac{1}{2}\pi m \left( \cos(\omega_0 - 2\omega_r)t - \cos(\omega_0 + 2\omega_r)t \right)$$

$$- \frac{2mI_m}{\pi(m^2 - 9)} \cos \frac{1}{2}\pi m \left( \cos(\omega_0 - 3\omega_r)t - \cos(\omega_0 + 3\omega_r)t \right)$$

$$\dots\dots\dots$$

It is seen that if the modulation index  $m = \Delta f/f_r$  is selected as

$$m = 2, 4, 6, \dots \text{ i.e. } \Delta f = 2f_r, 4f_r, 6f_r, \dots$$

the amplitude of the carrier vanishes. On the other hand the amplitudes of the sideband components vary proportionally to  $m/(m^2 - n^2)$  where  $n$  is the order of the sideband; plotting the quantity  $m/(m^2 - n^2)$  versus  $n$  (fig 2) it is found that the amplitudes of the sidebands are clustered in the vicinity of the order  $n$  which is closest to the modulation index  $m$ . More generally, an analysis of the spectral density function of random sequences of FSK signals has been developed by W.R. Bennett and S.O. Rice<sup>(6)</sup>; this function defines the distribution of the average signal power versus frequency and indicates the frequency bands of greatest interest. Two cases have been considered, one in which the phase is discontinuous (as would

occur when the frequency shift modulation is obtained by switching between two separate oscillators), and another in which the phase is continuous. Bennett and Rice have calculated some specific examples and plotted the spectral density function versus the frequency ratio  $(f - f_1)/f_r$  where  $f_1$  is one of the mark or space frequencies and  $f_r$  is the keying rate. A case of discontinuous phase is depicted in fig 3, with  $2\Delta f = 0.8 f_r$ ; it is noted that discrete spectral lines occur at the mark and space frequencies and a continuous spectrum falling off as  $1/f^2$  is present on either side of the passband. An example of continuous phase is illustrated in fig 4 (for  $2\Delta f = .8 f_r$ ) and in fig. 5 (for  $2\Delta f = 1.2 f_r$ ); no line spectra are found (unless there are degenerate relationships between  $f_1, f_2, f_r$ ) and a continuous spectrum falling off as  $1/f^4$  on either side of the passband is present.

Consideration of the spectral density function is important for the determination of the required passband of the antenna feed network. However, another problem of interest is that of overshoot or ringing which occurs because of phase distortion. In this connection, ringing is found to be larger where the cutoff is sharper, such as in the case of Chebychev or high order Butterworth responses. For these reasons it appears that Bessel type responses, which are characterized by flat delay over the passband, are more appropriate for the present application. Not only such filters exhibit lower overshoot phenomena, but they also yield lower pulse rise and decay times.



Synthesis of one-port antenna feed networks

For the purpose of the following calculations we assume the following design requirements:

Mark and Space frequencies:  $2 \times 10^4$  Hz and  $2.05 \times 10^4$  Hz.

Antenna static capacitance 0.0523  $\mu$ F

Antenna resonant frequency 36 kHz

Antenna radiation resistance 0.09 Ohm

Radiated power 138 kW

Antenna current 1238 ampere

Antenna base capacitance at  $2 \times 10^4$  Hz 0.1175  $\mu$ F

Antenna base resistance 0.13 Ohm

The antenna base capacitance is computed taking into account the inductance of the downleads, using the formula

$$C_1 = C_0 (f_{res}^2 - f^2) / f^2$$

The above antenna characteristics are similar to those of the Vanspolis antenna.

The first approach to the synthesis of feed networks to obtain FSK operation with a shift of 500 Hz is based on the utilization of one-port networks, connected in series with the antenna (fig.6); since the antenna is equivalent to the series combination of a capacitance and a small resistance, the reactance function including  $C_a$  has one of the following forms:

$$X(s)_1 = K \frac{(s^2 + w_1^2)}{s}$$

$$X(s)_2 = K \frac{(s^2 + w_1^2)(s^2 + w_3^2)}{s(s^2 + w_2^2)}$$

etc.

Expanding in series the above relationships one finds:

$$X(s)_1 = Ks + Kw_1^2/s$$

$$X(s)_2 = Ks + b/s + 2cs / (s^2 + w_2^2)$$

where

$$b = k w_1^2 w_3^2 / w_2^2$$

$$2c = K ( w_1^2 + w_3^2 - w_2^2 - (w_1^2 w_3^2 / w_2^2) )$$

Clearly  $X(s)_1$  corresponds to the conventional single tuning inductor case, where  $L = K$  and  $C_a = 1/kw_1^2$ ; on the other hand  $X(s)_2$  corresponds to a network having two zeros of transmission, respectively at  $w_1$  and at  $w_3$ , and so on. Following this approach it is possible to synthesize an antenna feed network having any number of transmission zeros; in practice, in order to reduce complexity and losses, the number of zeros should be maintained at a minimum.

Clearly in the case of FSK modulation the two-zero of transmission case is of interest. Canonic realizations are readily obtained by expanding  $X(s)_2$  either in partial fractions or in continuous fractions. Expansion in partial fractions yields:

$$X(s)_2 = Ks + bs + 2c / (s^2 + w_2^2)$$

which corresponds to the configuration of fig. 7, consisting of a series combination of an inductor and of a parallel resonant circuit. Expressing the network components in terms of the parameters  $K, w_1, w_2, w_3$  one finds:

$$L_1 = K$$

$$C_a = w_2^2 / k w_1^2 w_3^2$$

$$C_2 = 1/K (w_1^2 + w_3^2 - w_2^2 - w_1^2 w_3^2 / w_2^2)$$

$$L_2 = K (w_1^2 + w_3^2 - w_2^2 - w_1^2 w_3^2 / w_2^2) / w_2^2$$

Since  $K, w_1$  and  $w_3$  are prescribed, it is necessary to determine

a suitable value for the pole  $\omega_2$ ; this must satisfy the relationship

$$\omega_1 < \omega_2 < \omega_3$$

Two conditions may be considered, one in which  $\omega_2^2$  is taken as the average of  $\omega_1^2$  and of  $\omega_3^2$ , i.e.

$$\omega_2^2 = (\omega_1^2 + \omega_3^2) / 2$$

and the other in which  $\omega_2$  is selected as the geometric mean of  $\omega_1$  and of  $\omega_3$ , i.e.

$$\omega_2 = \sqrt{\omega_1 \omega_3}$$

As example, taking the latter condition one finds the following values for the network components:

$$L_1 = K \approx 1 / \omega_1 \omega_3 C_a$$

$$C_2 = \omega_1 \omega_3 C_a / (\omega_1 - \omega_3)^2$$

$$L_2 = (\omega_1 - \omega_3)^2 / \omega_1^2 \omega_3^2 C_a$$

Substituting the values of  $C_a$ ,  $\omega_1$  and  $\omega_3$  previously given one finds the following component values for the network of fig 7

$$L_1 \approx 0.527 \text{ mH}$$

$$C_2 \approx 200 \text{ pF}$$

$$L_2 \approx 0.527 \text{ pH}$$

It is seen that the inductance  $L_1$  has a value which corresponds to series resonance of the antenna capacitance at the frequency  $(\omega_1 \omega_3)^{1/2}$  and that the inductance  $L_2$  and the capacitance  $C_2$  form a parallel resonant circuit tuned at the same frequency. However, the inductance  $L_2$  is too small (yielding a low quality factor) and the capacitance  $C_2$  is too large for a practical realization. The difficulty may be resolved by recourse to a transformer as shown in fig. 8. In this case the primary

has inductance  $L_2$  as required by the design; the secondary is tightly coupled to  $L_2$  and has a larger inductance  $L'_2$  and a smaller capacitance  $C'_2$ , both resonating at the frequency  $(w_1 w_3)^{1/2}$ . As an example, letting  $C'_2 = 0.5 \text{ pF}$  one has  $L'_2 = 0.123 \text{ mh}$ .

Similarly expansion in continuous fractions of  $X(s)_2$  leads to the following result:

$$X(s)_2 = \frac{a'}{s} + \frac{1}{\frac{b'}{s} + \frac{2c's}{s^2 + w_2^2}}$$

The corresponding network realization is shown in fig. 9, where

$$\begin{aligned} w_2^2 &= 1/L_4 C_4 = w_1^2 + w_3^2 - w_1^2 w_3^2 / w_2^2 \\ L_3 &= (w_1^2 + w_3^2 - w_1^2 w_3^2 / w_2^2) / w_1^2 w_3^2 C_4 \\ L_4 &= \frac{w_2^2 (w_1^2 + w_3^2 - w_1^2 w_3^2 / w_2^2)}{w_1^2 w_3^2 C_4 (w_1^2 + w_3^2 - w_2^2 - w_1^2 w_3^2 / w_2^2)} \end{aligned}$$

Substituting the given antenna parameters one finds:

$$\begin{aligned} L_3 &= 0.527 \text{ mh} \\ L_4 &= 0.862 \text{ h} \\ C_4 &= 0.123 \text{ pF} \end{aligned}$$

It is noted that the values of the inductance  $L_4$  and of the capacitance  $C_4$  are highly impractical. However, as in the previous case, a network transformation may be introduced, by recourse to a transformer, such that the driving point impedance is conserved (fig. 10). Assuming that the transformer has unity coupling one finds the following relationships among the components of the two networks:

$$\frac{(L_3 + L_4)C_4}{L_3} = \frac{(L'_4 + L'_5)C'_4}{L'_3}$$

$$L_3 L_4 / (L_3 + L_4) = L'_3 L'_5 / (L'_4 + L'_5)$$

$$L_4 C_4 = L'_5 C'_4$$

Letting  $L_3 = L_3'$  one can select  $C_4'$  and  $L_5'$  so that the last relationship is satisfied and, at the same time, the values of these components are practically realizable.

In conclusion, it has been shown that it is possible to synthesize one-port antenna feed networks having two separate passbands, centered respectively at the mark and at the space frequencies. These networks are realizable and can be designed for any conventionally tuned VLF antenna, with minor modifications of the existing components.

#### Synthesis of two-port antenna feed networks

In addition to the use of one-port antenna feed networks having a prescribed driving point impedance it is also possible to obtain the desired broadbanding by recourse to a two-port network, which incorporates the antenna capacitance as its output element, and exhibits a transfer function corresponding to the desired bandwidth (fig 11). Transfer functions of interest are the Butterworth or maximally flat, the Chebychev or equal ripple, the Legendre or maximum fall-off, the Bessel or linear phase. In the case of FSK modulation the latter type of transfer function is preferable, because of lower overshoot and better values of rise and decay time in pulse operation.

In the following the synthesis of two port antenna feed networks is approached using either the "coupled resonator bandpass filter approximation" (4) or the "classical low-pass band-pass approximation". Although general design formulas will be derived, the particular case of the Bessel function response will be emphasized. An application of the coupled resonator method for the realization of Chebychev filters has been developed by Zverev and Blinichikoff (5).

The starting configuration is a low pass prototype of second order (fig 12); when this is designed for unity load resistance and unity radian bandwidth the circuit elements  $R_1$  and  $L$  can be expressed in terms of the capacitance  $C$  with the following relationships:

Butterworth

$$R_1 = \frac{\sqrt{2} C - 1}{C^2 - \sqrt{2} C + 1} \quad L = \frac{C}{C^2 - \sqrt{2} C + 1}$$

Chebyshev (letting  $k = \sqrt{1 + \varepsilon^2} + \varepsilon$  where  $\varepsilon$  is the ripple)

$$R_1 = \frac{\sqrt{2k(\varepsilon + 1)} C - k(\varepsilon + 1)}{(1 + \varepsilon k) C^2 - \sqrt{2k(\varepsilon + 1)} C + k(\varepsilon + 1)}$$

$$L = \frac{k(\varepsilon + 1) C}{(1 + \varepsilon k) C^2 - \sqrt{2k(\varepsilon + 1)} C + k(\varepsilon + 1)}$$

Bessel

$$R_1 = \frac{3C - 1}{3C^2 - 3C + 1} \quad L = \frac{C}{3C^2 - 3C + 1}$$

It should be noted that the Legendre case for  $n=2$  coincides with the Butterworth case for  $n=2$ .

The coupled resonator bandpass filter approximation is based on the use of a transformation of lossless LC ladder networks (Cauer first form) into an equivalent network containing only series inductances and impedance inverters; the inductances are converted to the band-pass case with the transformation

$$s' = (s^2 + \omega_0^2) / \Delta\omega$$

and the impedance inverters are synthesized, with sufficient approximation for narrow band filters, by recourse to T or  $\Pi$  configurations of inductances and capacitances. In the present case the final configuration of the band pass filter is shown in fig 13, where the load resistance  $R_a$  is the antenna base resistance and the series output capacitance is the antenna capacitance  $C_a$ .

In general the relationships between the elements of the low pass prototype and those of the bandpass approximation are:

### Source and load resistances

$$R_s = \frac{2\pi L_1 \Delta f}{q_1} \qquad R_a = \frac{2\pi L_2 \Delta f}{q_2}$$

where  $q_1$  and  $q_2$  are the normalized quality factors of the low pass prototype,  $q_1 = L/R_1$ ,  $q_2 = C$ .

### Inductances

$$L_1 = \frac{1}{(2\pi f_o)^2 C_I} \qquad L_2 = \frac{1}{(2\pi f_o)^2 C_{II}}$$

where  $C_I = C_{s1}C_{12} / (C_{s1} + C_{12})$ ,  $C_{II} = C_a C_{12} / (C_a + C_{12})$  are the mesh capacitances of the band pass filter, and  $f_o$  is the center frequency of the bandpass realization.

### Coupling capacitor

$$C_{12} = \sqrt{C_I C_{II}} / \left( \frac{\Delta f}{f_o} k_{12} \right)$$

where  $k_{12}$  is the normalized coupling coefficient of the low pass prototype,  $k_{12} = 1/\sqrt{LC}$ , and  $\Delta f$  is the bandwidth of the bandpass filter.

Since in the present application the elements  $R_a$  and  $C_a$  are prescribed, the design should be started by assigning a value to the coupling capacitor  $C_{12}$ ; from  $C_{12}$  there follow  $C_{II}$ ,  $L_2$  and  $q_2$ .

Now, applying the design equations for the desired type of low pass prototype response, one can calculate the values of  $R_1$  and  $L$  and those of  $q_1$  and of  $k_{12}$ . Finally, using the relationship expressing  $C_{12}$ , the value of the mesh capacitance  $C_I$  is computed; from  $C_I$  the values of  $L_1$  and of  $R_s$  follow.

Although for the present design of FSK modulation the Bessel type response is preferable, we have calculated the designs for the Butterworth, the Chebyshev and the Bessel cases. Taking for the capacitance  $C_{12}$  values from 0.5  $\mu F$  to 3.0  $\mu F$  we have found the values shown in Tables I, II and III, also summarized in the graphs of figs 14, 15, 16,

Table I - Butterworth response

$C_{12}$	$L_2$ mh	C	$R_1$	L	$q_1$	$k_{12}$	$C_{II}$ $\mu F$	$C_I$ F	$L_1$ mh	$R_s$ ohm	$C_{s1}$ $\mu F$
0.5	0.659	11.5	0.129	0.098	0.76	0.945	0.095	$14.75 \times 10^{-10}$	42.5	176	14.75
2.0	0.575	10	0.151	0.115	0.76	0.935	0.111	$198. \times 10^{-10}$	3.13	129	0.01
3.0	0.558	9.75	0.167	0.1185	0.71	0.935	0.113	$420 \times 10^{-10}$	1.5	67.2	0.04

Table II - Bessel Response

$C_{12}$	$L_2$ mh	C	$R_1$	L	$q_1$	$k_{12}$	$C_{II}$ $\mu F$	$C_I$ F	$L_1$ mh	$R_s$ $\Omega$	$C_{s1}$ F
0.5	0.659	11.5	0.092	0.0316	.342	1.67	.095	$46.2 \times 10^{-10}$	13.6	125	$46.2 \times 10^{-10}$
1	0.060	10.5	0.101	0.0347	.344	1.67	.105	$166 \times 10^{-10}$	3.78	34.5	$169 \times 10^{-10}$
2	0.568	9.9	0.108	0.0374	.345	1.65	.111	$614 \times 10^{-10}$	1.03	9.42	$623 \times 10^{-10}$



Table III - Chebyshev Response (1 dB ripple)

$C_{12}$	$L_2$ h	C	$R_1$	L	$q_1$	$k_{12}$	$C_{II}$ $\mu F$	$C_I$ F	$L_1$ mh	$R_s$ $\Omega$	$C_1$ F
0.5	0.659	11.5	0.130	0.131	1.01	.815	.095	11.1 $\times 10^{-10}$	56.8	176	11.1 $\times 10^{-10}$
1.0	0.60	10.5	0.115	0.142	1.23	.825	.105	40.7 $\times 10^{-10}$	15.5	39.6	40.7 $\times 10^{-10}$
2.0	0.568	9.9	0.123	0.153	1.24	.814	.111	150 $\times 10^{-10}$	4.25	10.8	150 $\times 10^{-10}$
3.0	0.558	9.75	0.124	0.155	1.25	.806	.113	316 $\times 10^{-10}$	2.03	5.12	311 $\times 10^{-10}$

Considering the case of the Bessel type response, it is seen that the optimum design is obtained in correspondence of values of  $C_{12}$  between 1 and 2  $\mu\text{F}$ ; slightly larger values (about 3  $\mu\text{F}$ ) are indicated for the Butterworth and the Chebychev (1 dB ripple) responses.

Finally, another method of synthesis of the bandpass filter consists of direct application of the low pass-band pass transformation to the low pass prototype of fig 12; a further transformation is required to convert the network into one having a series output capacitance and another transformation, based on the Norton equivalence, is utilized to obtain components which are practically realizable.

For the low pass prototype of fig. 12 the component relationships for a Bessel type response are as follows

$$R_1 = \frac{3CR_2^2 - R_2}{3C^2R_2^2 - 3CR_2 + 1}$$

$$L = \frac{CR_2^2}{3C^2R_2^2 - 3CR_2 + 1}$$

The band pass transformed network is shown in fig 17; however, in order that this network be applicable to antenna problem, its configuration must be modified to exhibit a series output capacitance. This is done with sufficient approximation for narrow band using the equivalence between the two networks of fig 18 at the center frequency of the pass band:

$$C_2 = C_s \frac{1}{1 + (\omega_0 R_s C_s)^2}$$

$$R_2 = R_s \frac{1 + (\omega_0 R_s C_s)^2}{(\omega_0 R_s C_s)^2}$$

In our application the antenna resistance is  $R_s = 0.18 \text{ Ohm}$  and the antenna capacitance is  $C_s = 0.1175 \text{ } \mu\text{F}$ . There follows

$$C_2 \cong C_s = 0.1175 \times 10^{-6} \text{ F} \quad R_2 = 24,500 \text{ Ohm}$$

Recapitulating, the design procedure for the low pass-band pass approximation method is as follows: in the normalized low pass prototype one has

$$R_2 = 1 \quad C = 0.117 \times 10^{-6} \times 25,400 \times 2\pi \times 10^4 = 9.39 \text{ F}$$

From these values one calculates  $R_1$  and  $L$ , i.e.

$$R_1 = 0.115 \text{ Ohm} \quad L = 0.0396 \text{ h}$$

In the corresponding band pass network one has

$$R_1 = 0.115 \text{ Ohm}, \quad L_1 = 1.25 \times 10^{-5} \text{ h}, \quad C_2 = 2.99 \times 10^{-3} \text{ F}$$

where the passband of 500 Hz and the load of 1 Ohm have been assumed. Denormalizing to the load  $R_2 = 24,500 \text{ Ohm}$  one finds:

$$R_1 = 2920 \text{ Ohm}, \quad L_1 = 317 \times 10^{-3} \text{ h}, \quad C_1 = 2 \times 10^{-10} \text{ F}$$

$$C_2 = 0.1175 \times 10^{-6} \text{ F}, \quad L_2 = 0.537 \times 10^{-3} \text{ h}, \quad R_2 = 24,500 \text{ Ohm}.$$

Difficulties would be encountered in the realization of the inductance  $L_1$  because of its large value; for this reason a network transformation consisting of the utilization of an ideal transformer or of a Norton equivalence is applied.

The use of an ideal transformer is shown in fig 19; in practice an autotransformer is used, tapping the inductor  $L_2$ . Assuming that the transformer ratio is 1:10, the tap is made at 1/10 of the turns. The elements  $R_1$ ,  $L_1$  and  $C_1$  are modified as follows

$$R_1 = 29.2 \text{ Ohms}, \quad L_1 = 3.17 \times 10^{-3} \text{ h}, \quad C_1 = 200 \times 10^{-12} \text{ F}$$

The application of the Norton equivalence is shown in fig 20; the combination of capacitance  $C_1 \times t^2$  and an ideal transformer of ratio 1:t is equivalent to the network of fig. 20, where a negative capacitance  $C_1(1-t)$  is present. Substituting in the network of fig 14, absorbing the negative capacitance within  $C_2 = 0.1175 \times 10^{-6} \text{ F}$  and

converting to the form of fig. 17 one obtains the final network of fig. 21; in this network, assuming that  $t = 10$ , one has

$$R_1 = 29.2 \text{ Ohm}, \quad L_1 = 3.17 \times 10^{-3} \text{ h}$$

$$C_y = 180 \times 10^{-12} \text{ F}, \quad C_x = 20 \times 10^{-12} \text{ F}$$

$$C_2 = 0.1175 \times 10^{-6} \text{ F}, \quad R_a = 0.18 \text{ Ohm}$$

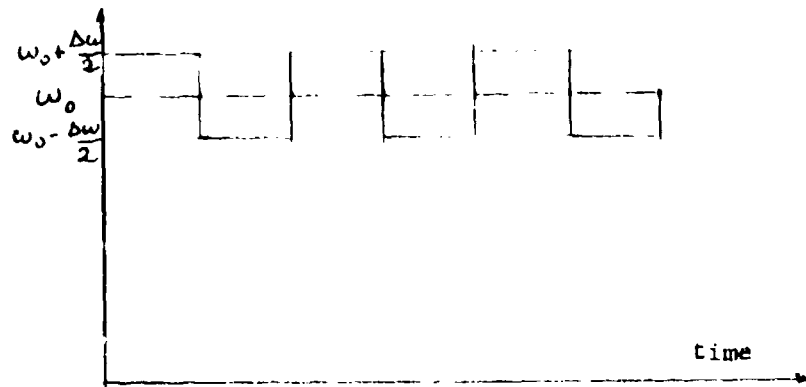
In general, once the calculations for the Norton equivalence have been completed, it is necessary to recalculate all the network elements in order to convert the output capacitance value to that of the antenna; however, in the present case, the modification required is negligible.

#### Conclusion

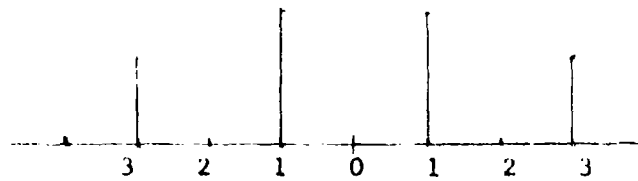
Several methods for the synthesis of antenna feed networks for operation with FSK modulation at wide frequency shift have been developed. Two types of networks have been derived, one of single-port type, having two zeros of transmission and allowing the antenna to operate with two separate bands centered respectively at the mark and at the space frequency, and the other of two-port type, exhibiting a continuous band which extends from the mark to the space frequency. Efforts have been made to develop networks which have economically realizable components and which can be applied to existing antenna installations without extensive modifications.

# References

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- 2 - H.G. Wolf - " High speed frequency keying of LF and VLF Radio circuits" IRE Trans. CS-5 (1957) N.3 pp 29-42
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- 6 - W.R. Bennett and S.O. Rice - "Spectral density and autocorrelation functions associated with binary frequency-shift keying" B.S.T.J. Sept. 1963 pp2355-2385.
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Example of FSK modulation



Sideband amplitudes for  $m = 2$

Fig. 1 - Example of spectrum of an FSK modulated wave

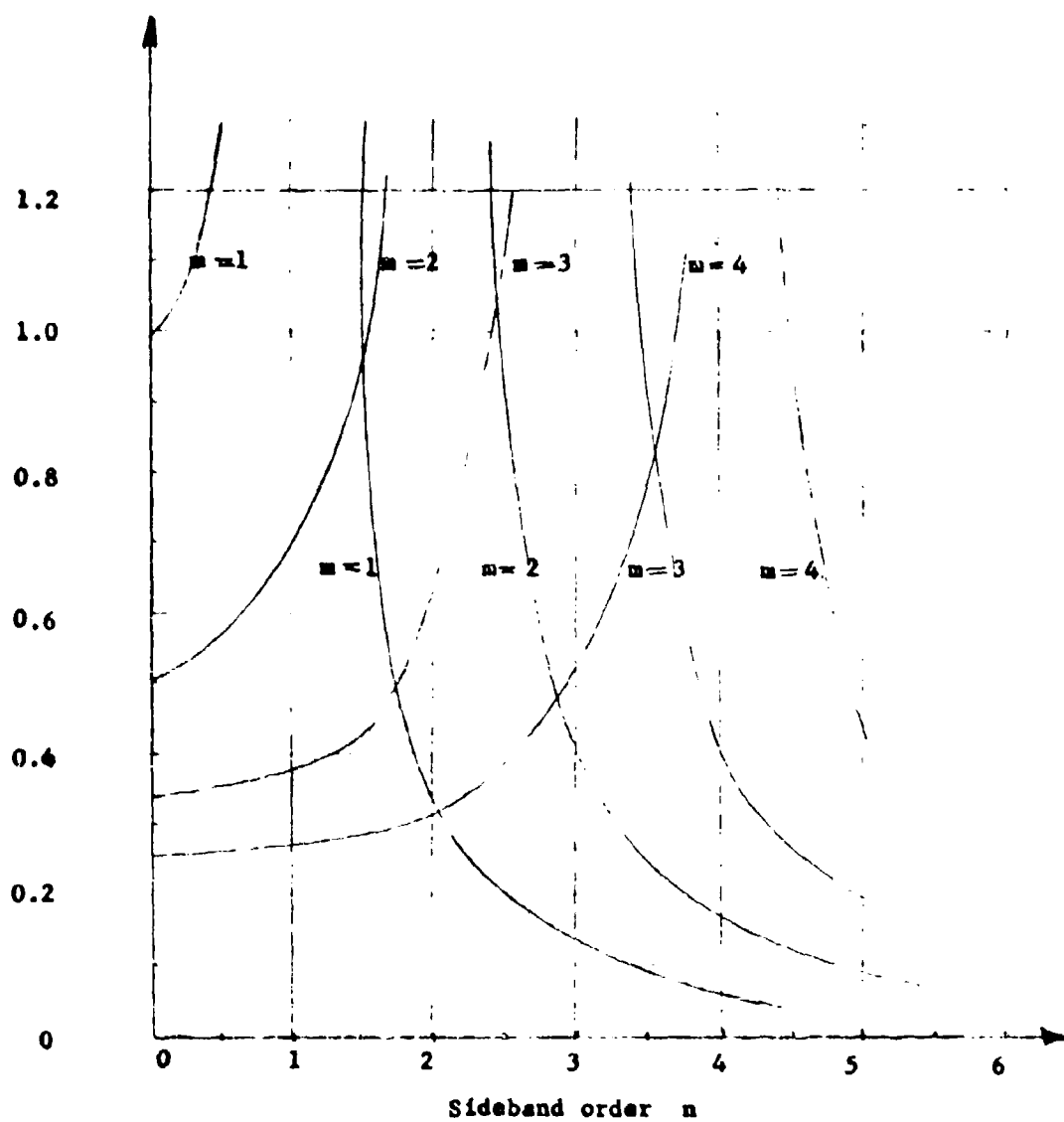


Fig 2 - Plot of  $m/(m^2 - n^2)$  versus  $n$

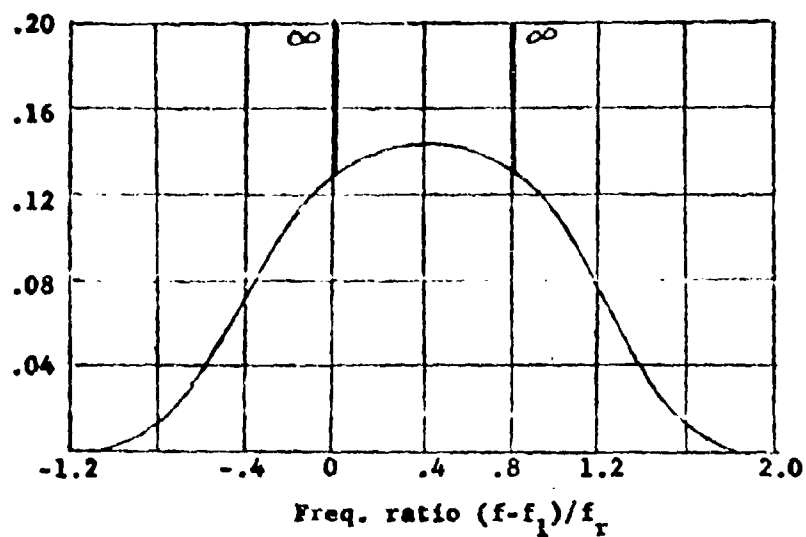


Fig. 3 - Spectral density of random FSK binary wave;  
discontinuous phase,  $2\Delta f = .8f_r$

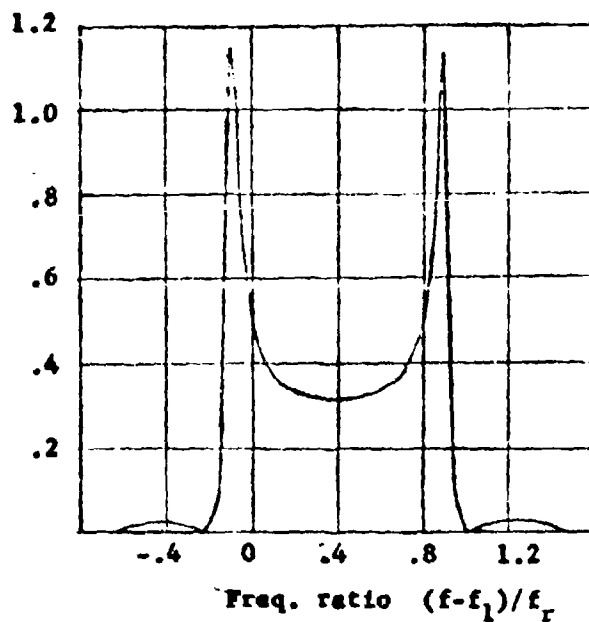


Fig. 4 - Spectral density of random binary FSK wave;  
continuous phase,  $2\Delta f = .8f_r$



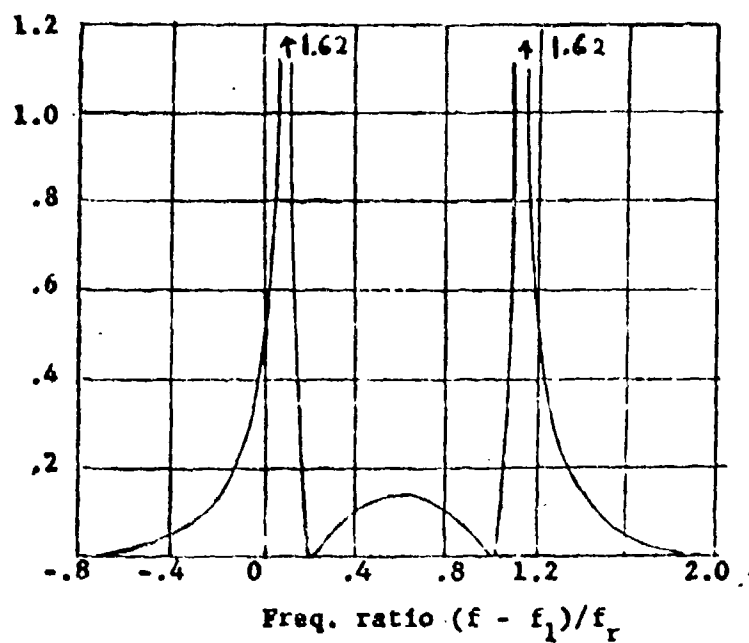


Fig.5 - Spectral density function of random binary FSK wave;  
-continuous phase,  $2\Delta f = 1.2f_r$

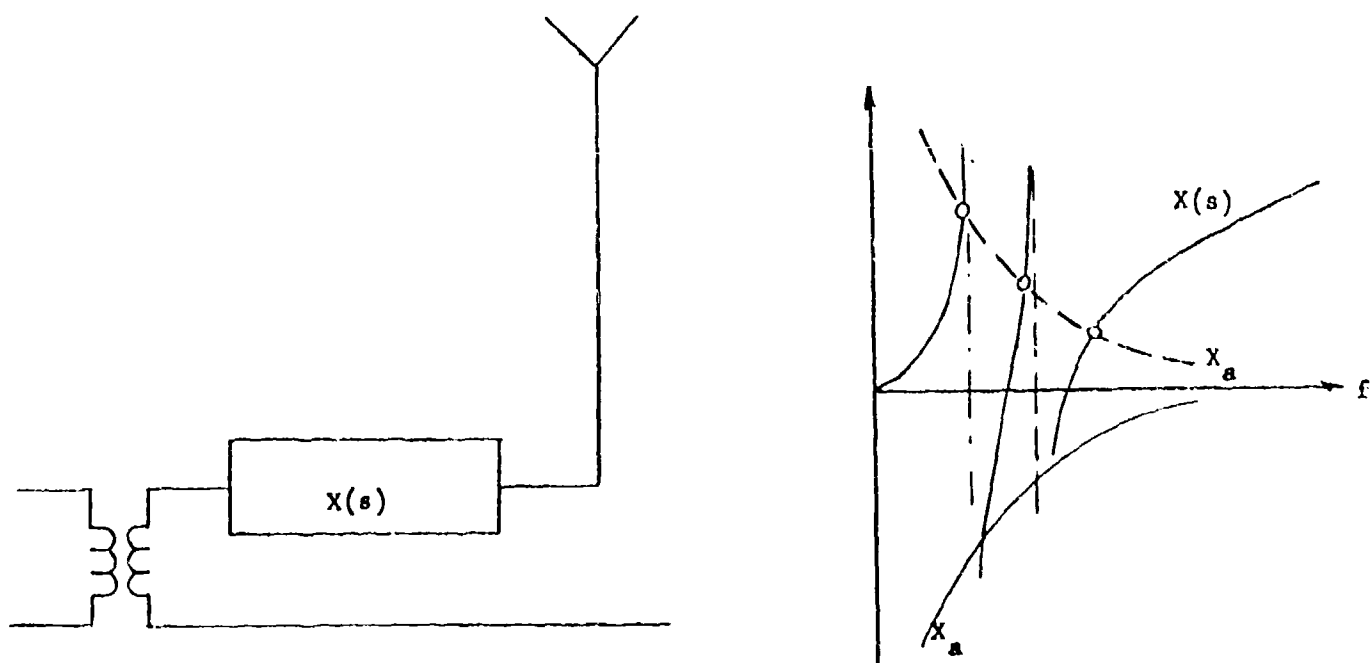


FIG. 6 - One-port antenna feed network

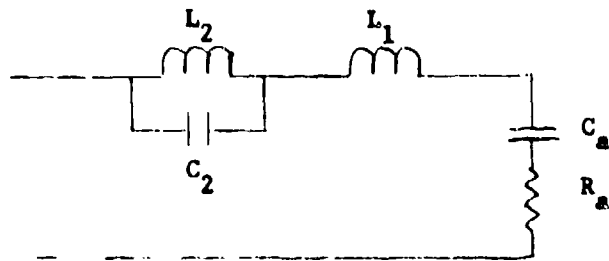


Fig 7 - Example of one-port feed network

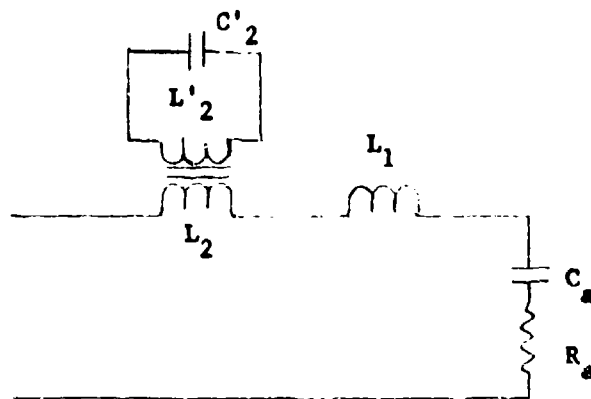


Fig 8 - Use of transformer to modify the network of fig 7

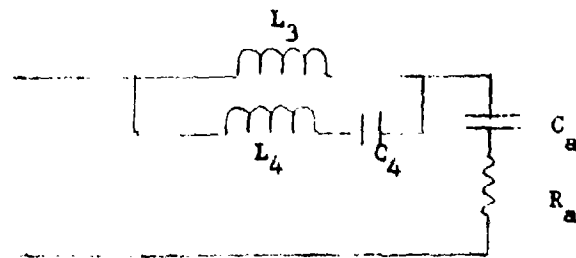


Fig 9 - Example of one-port feed network

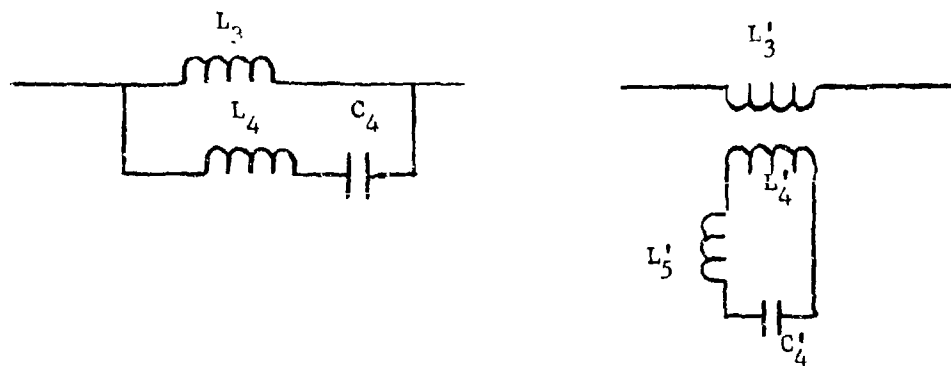


Fig. 10 - Equivalent network transformation

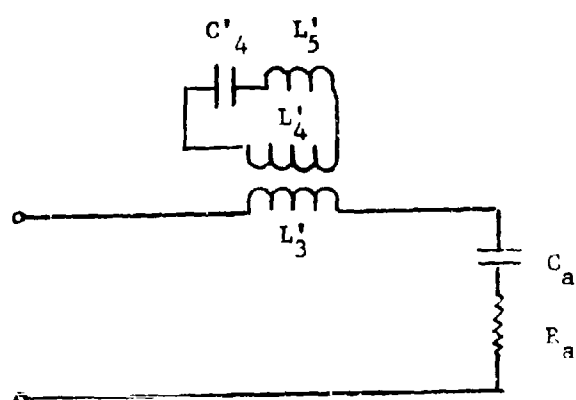
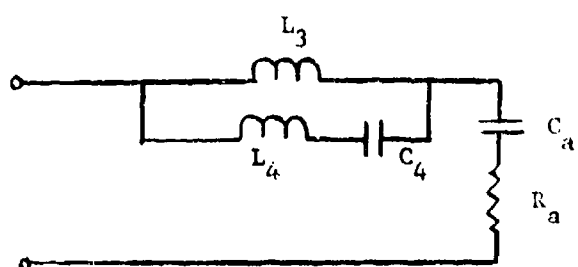


Fig.11 - Two passband filter for FSK modulation: a) synthesized network b) equivalent network

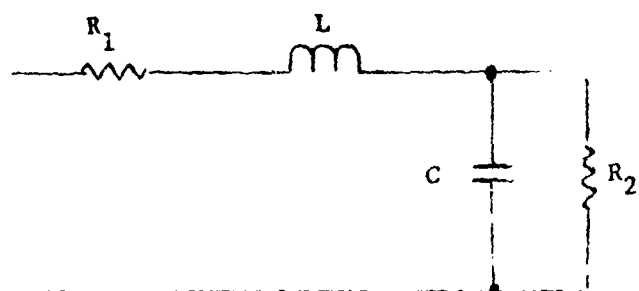


fig12. Low pass filter prototype

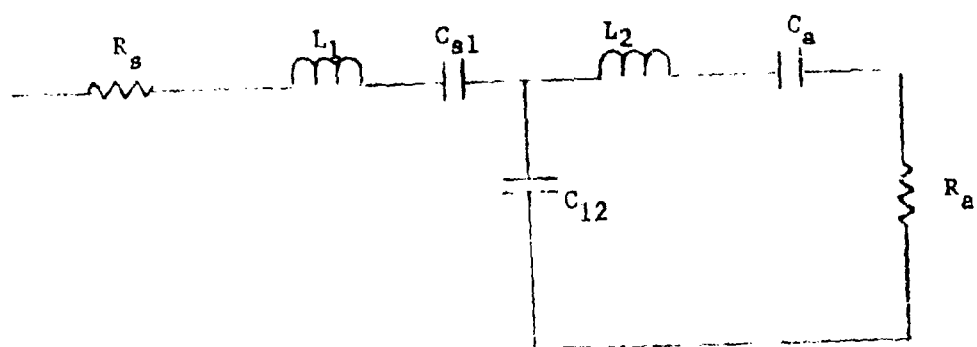


Fig 13 - Configuration of band-pass filter network

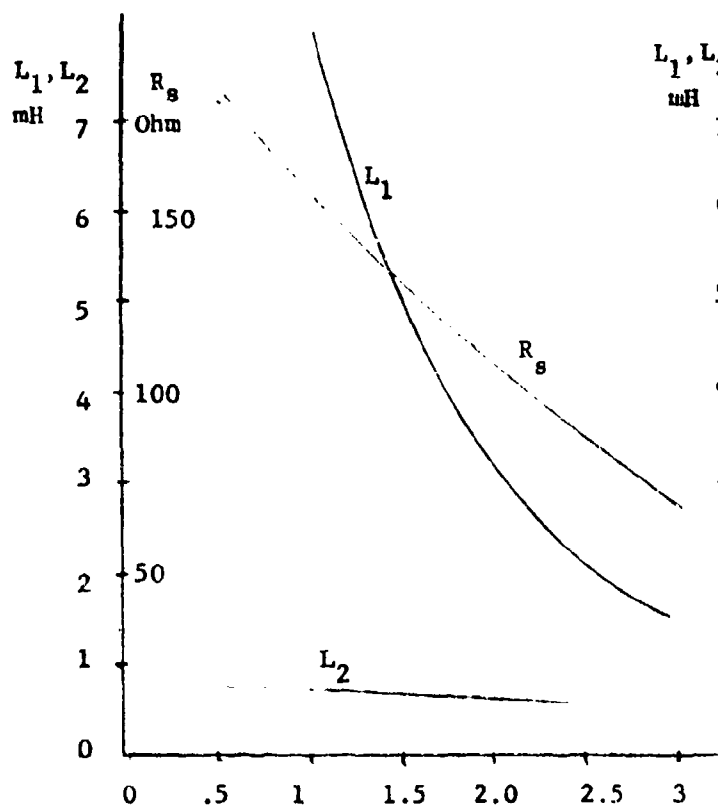


Fig14- Butterworth synthesis

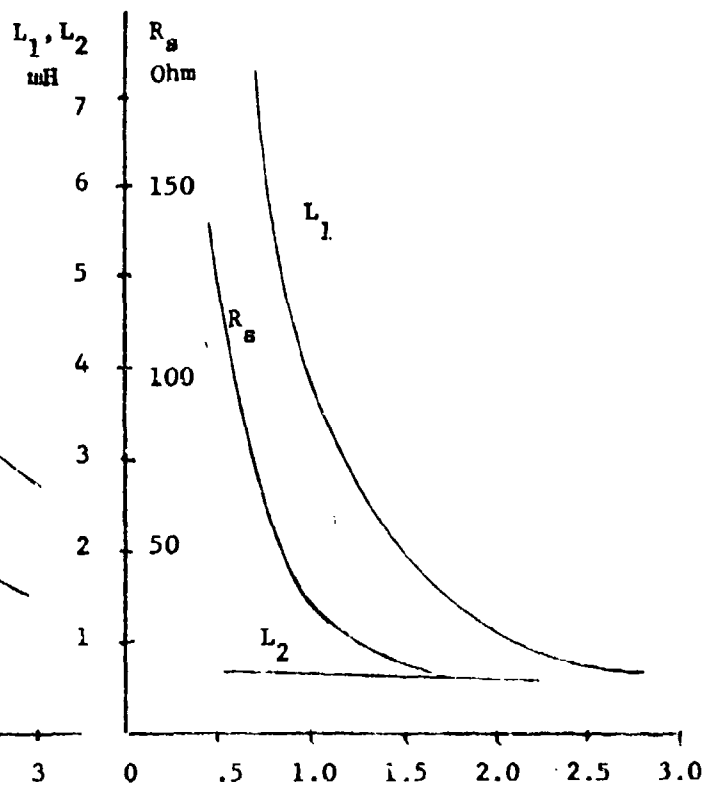


Fig15- Bessel synthesis

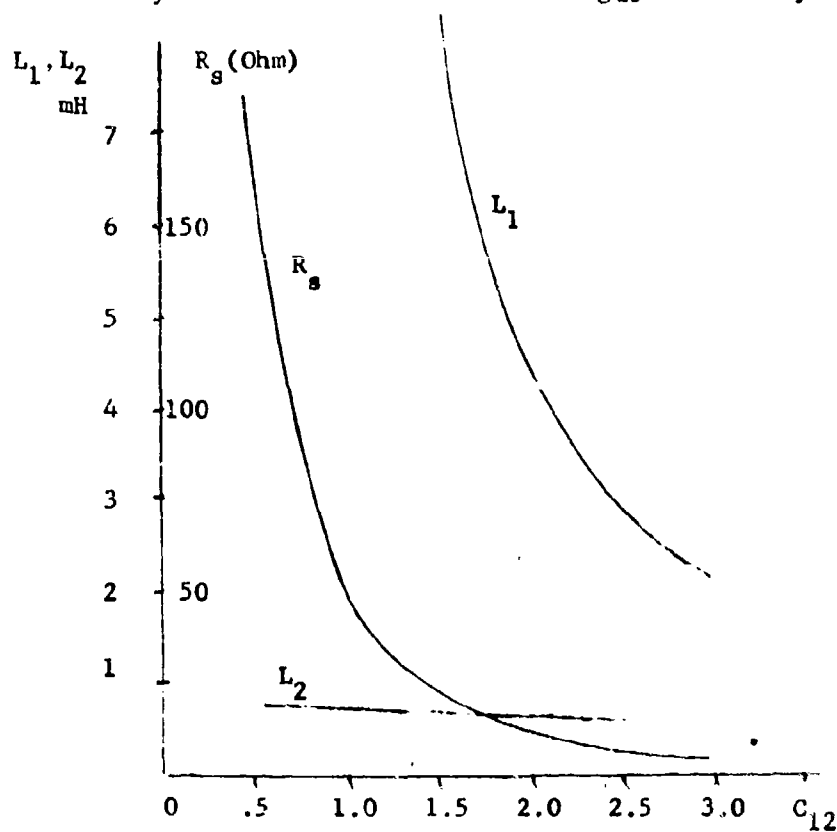


Fig16 - Chebyshev synthesis (1 dB ripple)

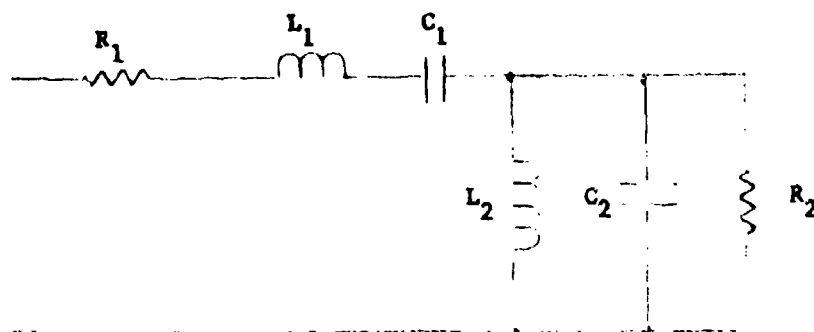


Fig. 17- Band pass filter corresponding to prototype of fig 12

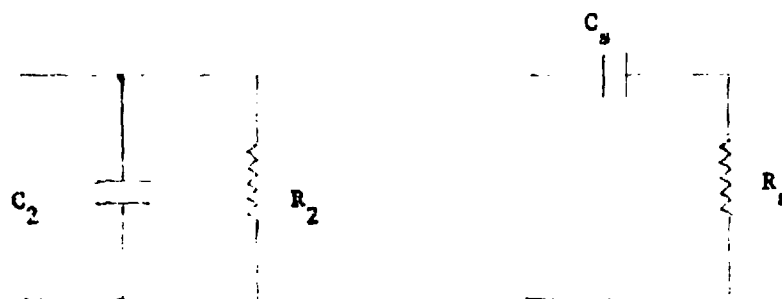


Fig 18- Equivalence between parallel combination  
and series combination of capacitance and resistance

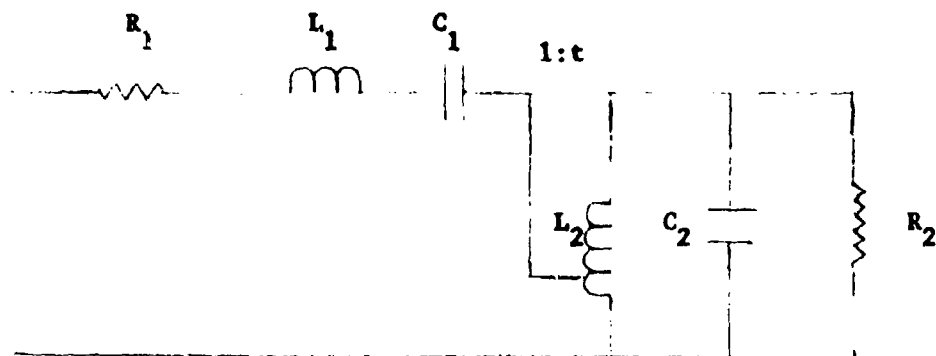


Fig. 19 - Band pass Bessel filter with ideal transformer

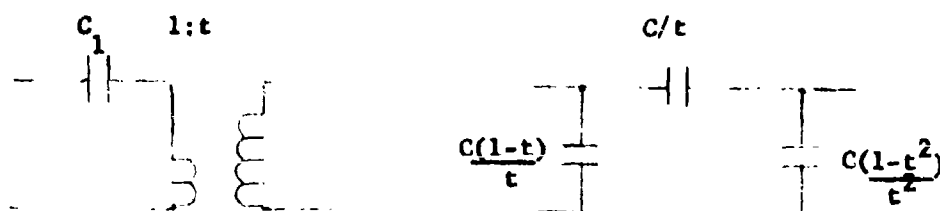


Fig.20- Example of Norton equivalent network transformation

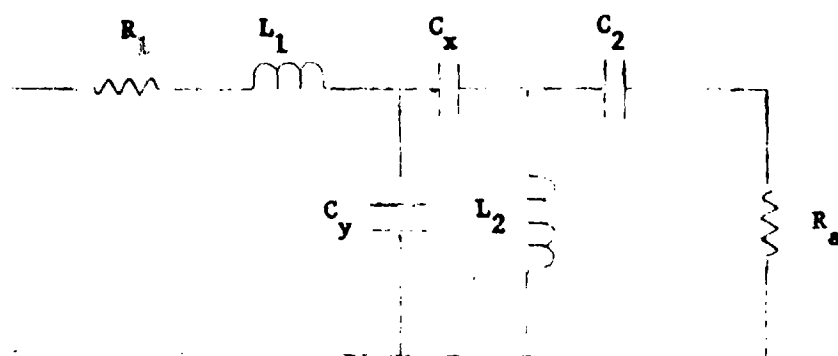


Fig.21 - Band pass Bessel filter with Norton transformation